Topic: Jump discontinuities

Question: Which of the following statements is true?

Answer choices:

A A jump discontinuity occurs when the right-hand limit and left-hand limit of a function are different but they both exist.

B A jump discontinuity occurs when the right-hand limit and left-hand limit of a function are different but only one exists.

C A jump discontinuity occurs when the right-hand limit and left-hand limit of a function do not exist.

D A jump discontinuity occurs when the right-hand limit and left-hand limit of a function are the same and they both exist.

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Solution: A

A jump discontinuity occurs when the right-hand limit and left-hand limit of a function both exist, but have different values.

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Topic: Jump discontinuities

Question: Choose the correct description of the jump discontinuity.

$$f(x) = \frac{x}{|x|}$$

Answer choices:

A The function has a jump discontinuity at $x = 1$.

B The function has a jump discontinuity at $x = -1$.

C The function has a jump discontinuity at $x = \infty$.

D The function has a jump discontinuity at $x = 0$.

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Solution: D

To solve for the discontinuities of the function, start by evaluating the right-hand and left-hand limits as x approaches 0. We need to use 0 for the limit because the function does not exist when $x = 0$.

For the right-hand limit, we'll use 3, 2 and 1.

$$\lim_{x \to 0^+} \frac{x}{|x|} = \frac{3}{|3|} = \frac{3}{3} = 1$$

$$\lim_{x \to 0^+} \frac{x}{|x|} = \frac{2}{|2|} = \frac{2}{2} = 1$$

$$\lim_{x \to 0^+} \frac{x}{|x|} = \frac{1}{|1|} = \frac{1}{1} = 1$$

For the left-hand limit, we'll use -3, -2 and -1.

$$\lim_{x \to 0^-} \frac{x}{|x|} = \frac{-3}{|-3|} = \frac{-3}{3} = -1$$

$$\lim_{x \to 0^-} \frac{x}{|x|} = \frac{-2}{|-2|} = \frac{-2}{2} = -1$$

$$\lim_{x \to 0^-} \frac{x}{|x|} = \frac{-1}{|-1|} = \frac{-1}{1} = -1$$

We can see that both one-sided limits exist. The right-hand limit is 1 and the left-hand limit is -1. Because both limits exist but aren't equal to each other, there's a jump discontinuity at 0.

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Topic: Jump discontinuities

Question: Choose the correct description of the jump discontinuity.

$$f(x) = \frac{x-1}{|x-1|}$$

Answer choices:

A The function has a jump discontinuity at $x = -1$.

B The function has a jump discontinuity at $x = 1$.

C The function has a jump discontinuity at $x = \infty$.

D The function has a jump discontinuity at $x = 0$.

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Solution: B

To solve for the discontinuities of the function, start by evaluating the right-hand and left-hand limits as x approaches 1. We need to use 1 for the limit because the function does not exist when $x = 1$.

For the right-hand limit, we'll use 4, 3 and 2.

$$\lim_{x\to 1^+} \frac{x-1}{|x-1|} = \frac{(4)-1}{|(4)-1|} = \frac{3}{|3|} = \frac{3}{3} = 1$$

$$\lim_{x\to 1^+} \frac{x-1}{|x-1|} = \frac{(3)-1}{|(3)-1|} = \frac{2}{|2|} = \frac{2}{2} = 1$$

$$\lim_{x\to 1^+} \frac{x-1}{|x-1|} = \frac{(2)-1}{|(2)-1|} = \frac{1}{|1|} = \frac{1}{1} = 1$$

For the left-hand limit, we'll use -2, -1 and 0.

$$\lim_{x\to 1^-} \frac{x-1}{|x-1|} = \frac{(-2)-1}{|(-2)-1|} = \frac{-3}{|-3|} = \frac{-3}{3} = -1$$

$$\lim_{x\to 1^-} \frac{x-1}{|x-1|} = \frac{(-1)-1}{|(-1)-1|} = \frac{-2}{|-2|} = \frac{-2}{2} = -1$$

$$\lim_{x\to 1^-} \frac{x-1}{|x-1|} = \frac{(0)-1}{|(0)-1|} = \frac{-1}{|-1|} = \frac{-1}{1} = -1$$

We can see that both one-sided limits exist. The right-hand limit is 1 and the left-hand limit is -1. Because both limits exist but aren't equal to each other, there's a jump discontinuity at -1.

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